THE JOURNAL OF
FINTECH &
DIGITAL ASSETS

JFDA, Volume 1(1), 2026

| **Research Paper**

Value-at-Risk (VaR) Based Portfolio Optimization and Risk Decomposition.

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Abstract
This study investigates portfolio optimization using the Value-at-Risk (VaR) as the risk measure to identify the sources of portfolio risk. Optimized portfolios under 95% and 99% VaR constraints were compared with an equal-weight portfolio, using a portfolio of 11 Standard & Poor's Depositary Receipt (SPDR) sector Exchange-Traded Funds (ETFs) representing the S&P 500 sectors (XLB, XLE, XLF, XLI, XLK, XLP, XLU, XLV, XLY, XLRE, XLC) over the period from 2018 to 2025. The delta-normal model was applied for VaR computation, and total portfolio risk was decomposed into marginal, component, and incremental VaR. Results show that VaR-based optimization enhances risk-return efficiency compared to the equal-weight benchmark, with minimal performance difference between 95% and 99% confidence levels. Risk decomposition further reveals that portfolio risk is highly concentrated in technology (XLK), consumer staples (XLP), and utilities (XLU) sectors, with XLK contributing the most to total portfolio risk. This suggests that optimization tends to favor a mix of growth and defensive assets. Overall, the findings highlight the trade-off between diversification and efficiency in risk-based portfolio construction. This study underscores the practicality of using VaR in portfolio optimization and risk attribution, and future research may explore extensions using conditional VaR (CVaR) or alternative market regimes to capture extreme risk dynamics more accurately.

Keywords: Value-at-Risk (VaR), marginal VaR, component VaR, incremental VaR, portfolio optimization.
JEL Classification: G10, G11, C50.

Article Information
Received: 02 February 2026.
Revised: 11 March 2026.
Accepted: 17 March 2026.
Published: 22 May 2026.

Citation:
Zhang, J. (2026). Value-at-risk (VaR) based portfolio optimization and risk decomposition. *The Journal of FinTech and Digital Assets*, 1(1), 26–44.

1. Introduction

Identifying and quantifying portfolio risk sources is crucial for effective risk management and capital allocation. For institutional investors and portfolio managers, understanding where risk originates within a portfolio helps guide asset allocation decisions, risk budgeting, and regulatory capital management. In particular, decomposing portfolio risk enables investors to identify dominant risk drivers and evaluate how individual assets contribute to overall downside exposure. Traditionally, variance and the standard deviation and variance have been widely used as measures of portfolio risk within Markowitz's mean-variance framework. However, a well-known limitation of variance is that it penalizes both upside and downside fluctuations symmetrically, treating both favorable and unfavorable deviations as risk, which may not accurately reflect investors' primary concern with potential losses. Moreover, variance as a risk measure was inadequate due to its assumption that returns are normally distributed (Ayodeji & Ingram, 2015). In practice, financial returns often deviate from the normal distribution and frequently exhibit fat-tailed distributions (Hallerbach, 1999), making variance-based risk measures less informative in capturing extreme downside events.

In the 1980's, Value-at-Risk (VaR), an alternative risk measure, emerged as a concept and grew popular in the early 1990's owing to J.P. Morgan. VaR indicates the greatest potential loss of a portfolio at a given confidence level over a defined time horizon (Tardivo, 2002), which is considered the financial industry's standard for measuring exposure to financial price risks (Dubofsky & Miller, 2003). Contrary to variance, VaR has the advantage of focusing solely on downside outcomes; its capability in addressing worst-case scenarios makes it appealing for investors, regulators, and risk managers who are more concerned with catastrophic losses.

However, the non-coherent property of VaR is inherently problematic for modeling financial risks, as the risk of the overall portfolio is not simply the total sum of the individual risks (Duc et al., 2018). For example, despite the positive expected returns of each portfolio, at 95% confidence, an investor of this portfolio is still subjected to a risk of -1.9% returns (Duc et al., 2018). Relying on VaR may not help build an optimal portfolio in terms of investment selection. Given that, conditional Value-at-Risk (CVaR) came up by Rockafellar and Uryasev (2000) as an alternative measure that considers the losses that will be held when the VaR threshold is exceeded. Different from the VaR, CVaR is a coherent risk measurement (Pflug, 2000), which propelled its popularity as an efficient measure of risk. CVaR is considered superior to VaR in portfolio optimization (Sarykalin et al., 2008). Nonetheless, VaR remains widely used by institutions and regulators, and it is valuable to understand the implications of using VaR directly in asset selection. In this paper, the focus will be solely on the VaR.

However, VaR does not identify how individual assets contribute to the total; it only measures the overall portfolio risk. An efficient method of decomposing overall risk will then be a useful tool for managing portfolio risk, which allows the risk managers to select assets that provide the best risk-return trade-off to individual risk factors (Yamai & Yoshiba, 2002). Hallerbach (1999) developed a method of decomposing VaR into individual factors - marginal VaR, component VaR, and incremental VaR. The risk decomposing technique can identify the 'hot spots' of risk in the portfolio, enabling effective risk budgeting and better-informed asset allocation (GARP, 2024), which will contribute to constructing an optimal minimum-risk portfolio. The results will be beneficial for the financial institutions or individual investors who need to mitigate effectively against the potential risks. Effective risk management would offer a platform for future policy and strategic decision-making (Duc et al., 2018).

The Sharpe Ratio is a classic metric for risk-adjusted performance, which is defined as expected excess return divided by standard deviation. An alternative is the Reward-to-VaR Ratio, introduced by Alexander and Baptista (2003), which is defined as the expected portfolio excess return divided by the portfolio VaR at a given confidence level. Thus, this measure evaluates portfolio performance by scaling expected excess return with VaR, thereby explicitly incorporating downside risk into the efficiency criterion. Although this measure is closely related to the Sharpe Ratio, it is particularly attractive for investors concerned with extreme losses, since VaR captures tail risk rather than symmetric volatility.

Inspired by this concept, this project adopts a Return/VaR ratio as the optimization objective. This measure is essentially equivalent to the Reward-to-VaR Ratio, with the simplifying assumption of a zero risk-free rate. By maximizing the ratio of expected portfolio return to portfolio VaR, the optimization framework ensures that asset allocations are chosen to maximize reward relative to downside risk exposure. Thus, this paper aimed to 1) construct portfolio that maximize the Sharpe Ratio under VaR-based risk constraints, thereby demonstrating the role of VaR in risk management; 2) compute and analyze marginal, component, and incremental VaR in order to decompose overall portfolio risk and identify the risk sources, which will provide insights into the relative contributions of individual assets and help identify the main drivers of portfolio risk under different market conditions.

To achieve these objectives, an optimal long-only sector ETF portfolio was constructed, and results between 95% and 99% VaR confidence levels were compared to examine how a stricter tail-risk constraint affects asset selection. A full VaR decomposition was then conducted, including marginal VaR - the sensitivity of portfolio VaR to a marginal increase in asset weight, component VaR - each asset's actual contribution to total portfolio VaR, and incremental VaR - the change in portfolio VaR if an asset is removed. This bridges the gap between abstract risk-adjusted performance measures and practical portfolio construction, illustrating how optimizing for Return/VaR can shift the portfolio toward safe and more efficient territory in terms of downside risk.

The remainder of this study is organized as follows: Section 2 reviews relevant literature on VaR and risk decomposition. Section 3 describes the dataset and asset selection criteria, and outlines the methodology, including definitions of risk measures and the optimization approach. Section 4 presents the empirical results, including the optimal portfolios under 95% and 99% VaR, their performance, and a detailed risk decomposition analysis. Finally, Section 5 concludes the study with a discussion of the findings and their implications for risk-focused portfolio management.

2. Literature Review

Research integrating VaR into portfolio optimization builds on the recognition that investors may care more about downside extremes than average volatility. Roy (1952) first introduced the "safety-first" criterion, focusing on minimizing the probability of disaster falling below a certain threshold. This idea had then evolved into downside risk measures and influenced the later development of VaR. By the late 1990s, VaR had become a standard risk metric in the finance industry and was popularised by the release of RiskMetricsTM by J.P.Morgan in October 1994.

The greatest advantage of VaR is that it summarizes risk in a single and understandable number (Horcher, 2011). In other words, VaR is the maximum amount of money that can be lost in portfolio investment, over a specific time horizon window and a given confidence level (Koziorowska, 2012). This definition highlights VaR's focus on tail risk by providing a clear cut-off point that separates typical portfolio fluctuations from extreme loss outcomes. For example, a 99% VaR calculated over a 10-day holding period implies that the maximum portfolio loss over the next 10 days is expected to be exceeded with a probability of only 1%.

2.1. Value at Risk - Delta Normal Model

VaR can be calculated using the methods of historical, Monte Carlo, Risk Metrics, and Variance-Covariance approach (Duc et al., 2018). These methods differ in terms of market data needs, computational demands, and modelling capacity for diverse instruments (Koziorowska, 2012). Historical and Monte Carlo simulation methods are preferred for non-linear portfolios such as options. However, these methods are not intended to be discussed as this paper focuses on ETFs only. The variance-covariance approach (also known as the parametric approach) is the most used and the closest one to the definitions and concepts from the Modern Portfolio Theory, as it expresses the VaR as a multiple of the portfolio deviations (Tardivo, 2002). This approach is computationally efficient, as it only requires matrix multiplication. By approximating each position with its linear exposure, the approach can be applied even to a portfolio with a large number of assets and updated in real time as positions change (Jorion, 1997).

The Delta Normal model, one of the main variance-covariance models, was originally introduced by J.P. Morgan in the RiskMetrics document and will be adopted in this project to analyse the volatility of the returns of the market factors. The risk of a portfolio, quantified through multivariate statistical techniques, depends on both the volatility and the correlation (Tardivo, 2002). This model exactly defines the relations among the financial positions and the market risk factors (e.g., the exchange quotations, rates, and shares) (Tardivo, 2002). It assumes that asset returns are normally distributed, allowing volatilities and correlations to be calculated directly from the underlying time series (Koziorowska, 2012). Under this approach, the expected return and standard deviation of a portfolio are expressed as:

$$\mu_p = \sum_{i=1}^{n} x_i \overline{R_i}, \tag{1}$$

$$\sigma_p = \sqrt{x'\Sigma x}, \tag{2}$$

where x_i represents the value of the i-th asset, $\overline{R_i}$ its expected return, x the portfolio weight vector, and Σ the covariance matrix.

The VaR for an individual financial instrument can then be defined as:

$$VaR_i = -(\mu - \alpha\sigma w_i), \tag{3}$$

where μ denotes the expected return, σ the standard deviation of returns, w_i is the value of the i-th asset, and α is the quantile corresponding to the chosen confidence level (e.g., $\alpha = 1.65$ for 95% and $\alpha = 2.32$ for 99%) (Koziorowska, 2012). Assuming $\mu = 0$, then the VaR equation simplifies to:

$$VaR_i = \alpha\sigma w_i. \tag{4}$$

The confidence range describes the probability level with which a loss in a specific time horizon, its choice depends on the aversion level to the risk (Tardivo, 2002).

2.2. Risk Decomposition

Portfolio optimization aims to allocate limited capital efficiently in order to achieve the desired objectives, subject to decision variables, the objective function, and constraints (Ayodeji & Ingram, 2015). Introducing VaR as a shortfall constraint into the portfolio optimization is more consistent with individuals' intuitive perception of risk, as this approach allows the investors or the portfolio managers to focus on the probability of portfolio value falling below a specific VaR threshold (Campbell et al., 2001). In other words, the goal of portfolio optimization is to find optimal solutions that allocate more resources to different assets in a portfolio that investors believe are less risky or that will yield the highest return with respect to some constraints (Ayodeji & Ingram, 2015). Among different categories of risk, market risk is of primary concern, as it arises from fluctuations in market prices influenced by factors such as interest rates, stock indices, and foreign exchange rates, etc. (Ayodeji & Ingram, 2015). Recent studies continue to examine VaR-based portfolio risk management and risk-return evaluation frameworks, highlighting their practical value in asset allocation and financial risk monitoring (Campbell et al., 2001; Huisman et al., 1999; Jiménez et al., 2020; Leduc & Perera, 2025; Tsao & Liu, 2006)

However, as mentioned above, a single VaR only provides the overall portfolio risk and does not indicate how individual assets contribute to that risk. Over time, risk managers refined VaR not only as a measure of total exposure but also to identify sources of risk and set appropriate limits in order to improve portfolio allocation (Koziorowska, 2012). The practical challenge lies in determining which portfolio elements should be adjusted or replaced to make VaR more effective. To operationalise this refinement, VaR can be decomposed into marginal VaR, component VaR, and incremental VaR, which provides insights into how individual assets contribute to overall portfolio risk.

2.2.1 Marginal VaR

Marginal VaR (MVaR) measures the sensitivity of the total portfolio VaR to a small change in the position of a particular asset. In other words, it captures how the overall portfolio risk would change if the weight of one asset were increased or decreased infinitesimally. Formally, it is defined as the partial derivative of the portfolio VaR with respect to the exposure of the asset i (Dowd, 2007):

$$MVaR_i = \frac{\partial VaR}{\partial x_i}. \tag{5}$$

From a practical perspective, MVaR can be expressed as a function of the covariance between the return of asset i (R_i) and the portfolio return (R_p):

$$MVaR_i = \alpha \frac{\mathrm{cov}(R_i,R_p)}{\sigma_p}, \tag{6}$$

where α is the quantile of the standard normal distribution corresponding to the chosen confidence level, and σ_p is the standard deviation of the portfolio return.

Furthermore, MVaR is closely related to the asset's beta coefficient with respect to the portfolio:

$$\beta_i = \frac{\mathrm{cov}(R_i,R_p)}{\sigma_p^2} = \rho_{ip}\frac{\sigma_i}{\sigma_p}, \tag{7}$$

where ρ_{ip} is the correlation coefficient between the asset i and a portfolio p.

The beta coefficient is the basis for the capital asset pricing model (CAPM) developed by Sharpe (1964), describing the relation of its returns with those of the financial market. It can be estimated for individual companies using regression analysis against a stock market index (Koziorowska, 2012).

Finally, by linking to the total portfolio, MVaR is therefore expressed as:

$$MVaR_i = \alpha(\beta_i \cdot \sigma_p) = \frac{VaR}{W} \cdot \beta_i, \tag{8}$$

where W is the total value of the portfolio.

Thus, MVaR reflects the marginal contribution of asset i to portfolio risk. This measure provides useful information for portfolio managers by identifying which assets exert the largest marginal effect on total risk. For example, to lower the portfolio VaR, the investors or managers should pick the asset with the largest MVaR as its greatest hedging effect (GARP, 2024).

2.2.2 Incremental VaR

Incremental VaR (IVaR) quantifies the change in total portfolio VaR when a new position is added. It measures the discrete effect of trading decisions on portfolio risk. Formally, IVR is defined as the difference between the VaR of the portfolio with the new position and the VaR of the initial portfolio p (Dowd, 2007):

$$IVaR = VaR(p + a) - VaR(a), \tag{9}$$

where $VaR(p)$is the VaR of the original portfolio, and VaR$(p + a)$ is the VaR after including a new position a. Alternatively, for an approximation in the case of large portfolios where a full revaluation requires a large number of computations, IVaR can be estimated using MVaR (Koziorowska, 2012):

$$IVaR = (MVaR)' \cdot a. \tag{10}$$

This simplified VaR method has been proven to perform well for large portfolio especially where a proposed trade is likely to be of a relatively small size, and thus it allows real-time trading limits (GARP, 2024).

A positive IVaR indicates that the new position increases the portfolio risk, while a negative IVaR implies that the new trade is risk-reducing and functions as a natural hedge against the existing portfolio (Koziorowska, 2012). This hedging effect, however, is limited to a certain range of position sizes. To determine optimal hedging strategies, when a new position a is added to a single risk factor i, the portfolio variance of dollar returns can be expressed as (GARP, 2024):

$$\sigma_{p+a}^2 W_{p+a}^2 = \sigma_p^2 W^2 + 2aW\sigma_{ip} + a^2\sigma_i^2, \tag{11}$$

where σ_{ip} is the covariance between the asset i and the portfolio p.

Differentiating with respect to a yields the condition for the variance-minimising position:

$$a^* = -W\frac{\sigma_{ip}}{\sigma_i^2} = -W\beta_i\frac{\sigma_p^2}{\sigma_i^2}. \tag{12}$$

This solution represents the best hedge, i.e., the additional amount to invest in an asset i that minimises the overall portfolio risk. The concept of best hedge illustrates how IVaR can be used not only for risk measurement but also for designing effective hedging strategies within portfolio management.

2.2.3 Component VaR

Component Value at Risk (CoVaR) indicates how the portfolio VaR would change if the component were deleted. It decomposes total portfolio VaR into additive contributions from individual assets. By construction, component VaRs sum to the portfolio VaR. Unlike simply summing up stand-alone VaRs, which neglects diversification benefits, CoVaR also captures the power of diversification (GARP, 2024). Thus, CoVaR for an asset i was defined as (Koziorowska, 2012):

$$CoVaR_i = w_i \frac{\sigma VaR}{\sigma w_i}, \tag{13}$$

$$VaR = \sum_{i=1}^{n} CoVaR_i, \tag{14}$$

where w_i is the portfolio weight of the asset i.

Moreover, CovaR can be computed via MVaR:

$$CoVaR_i = w_i W \cdot MVaR_i, \tag{15}$$

where w_i is the portfolio weight of the asset i, W is the portfolio value.

This formulation implies that CoVaR represents the risk contribution of the current dollar position of asset i to the portfolio's total VaR. However, this decomposition is more useful with large portfolios containing many small positions, as the quality of this linear approximation improves when the VaR components are small (GARP, 2024). In a nutshell, assets with negative CoVaR act as natural hedges, while those with positive CoVaR increase the overall portfolio risk. Hence, CoVaR provides investors or risk managers with a practical tool for identifying key risk contributors and guiding portfolio rebalancing decisions. In sum, MVaR and CoVaR are extremely useful tools for identifying risk sources, finding natural hedges, defining risk limits, reporting risk, and improving portfolio allocations (Koziorowska, 2012).

These three VaR-based risk decomposition tools are closely related and provide complementary perspectives on how individual assets contribute to overall portfolio risk (Koziorowska, 2012). MVaR captures the sensitivity of portfolio VaR to an infinitesimal change in an asset's position, reflecting the local slope of the VaR function with respect to asset weights. In this sense, MVaR measures how small adjustments in each asset's allocation affect total portfolio risk. CoVaR extends this concept by quantifying the contribution of each asset to total portfolio VaR as the product of the asset's portfolio weight and its marginal VaR. CoVaR therefore provides an additive decomposition of portfolio risk, allowing total VaR to be allocated back to individual assets in proportion to their risk contributions. IVaR, in contrast, measures the discrete change in portfolio VaR resulting from the complete removal of an asset from the portfolio. IVaR, thus, reflects the overall risk impact of including a particular asset, capturing both marginal sensitivity and portfolio interactions. Together, these three measures offer a comprehensive framework for understanding portfolio risk structure.

MVaR highlights marginal sensitivity, CoVaR enables additive risk attribution, and IVaR captures the total effect of asset inclusion or exclusion.

In summary, the literature review suggests VaR-based optimization is inherently aligned with investors' concern for tail risks. This paper provides an empirical case study on implementing VaR within portfolio optimization, emphasizing the application of three VaR decomposition tools. Together, these elements aim to bridge the gap between theoretical portfolio optimization and practical risk management.

3. Data and Methodology

The daily close prices of selected 11 Standard & Poor's Depositary Receipt (SPDR) sector ETFs - materials (XLB), energy (XLE), financials (XLF), industrials (XLI), technology (XLK), consumer staples (XLP), utilities (XLU), health care (XLV), consumer discretionary (XLY), real estate (XLRE), and communication services (XLC) between 1st January, 2015 and 22nd August, 2025 were downloaded from Yahoo Finance. To reflect realistic portfolio construction, long-only constraints, and a full investment requirement ($\sum w_i = 1$) were imposed. No leverage or short selling is typical for many investment funds.

Sector-focused ETFs allow us to observe how different portions of the equity market contribute to portfolio risk. It also limits the impact that any single stock's extreme move may have on a sector ETF. Thus, using sector ETFs is ideal for a sector-level risk decomposition. Besides, these stocks selected collectively represent all major sectors of the S&P 500, providing a well-diversified and comprehensive market coverage, covering cyclical sectors (e.g., technology and financials) and defensive sectors (e.g., staples and utilities), which is ideal for analyzing how a tail-risk-focused optimization allocates between high-risk/high-return and low-risk/low-return assets. Additionally, the ETFs' long trading histories with sufficient liquidity reduce data quality issues. A ten-year period ensures robustness across diverse market conditions, including bull and bear markets, especially the COVID-19 shock.

However, the actual data time-window acquired spans from 19th June 2018 to 22nd August, 2025, since the XLC sector was launched in June 2018. To maintain a consistent comparison across all 11 sectors, the effective sample period to the same start date was restricted. This means roughly 7 years of daily data (1,805 observations/trading days) were used, which is sufficient for our analysis. Log daily returns were used and computed from the close prices, a transformation commonly adopted due to its ability to generate a return distribution that better conforms to the normality assumption underlying VaR models. The obtained data was checked for quality to make sure a complete dataset without missing values.

The basic statistics, including count, mean, std, min, 25%, 50%, 75%, and max, were summarized in Table 1 and showed substantial cross-sector differences in mean and volatility (std). Over the 2018-2025 period, the average daily return ranged from 0.03%, mainly in the defensive sectors, to 0.08% (XLK, the technology sector). The volatilities spanned from 1.00% (XLP) to 2.09% (XLE). As expected, the defensive sectors are less volatile, and the cyclical sectors are more volatile.

Table 1. Descriptive Statistics of 11 Sector Daily Returns

	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	XLRE	XLC
Mean	0.03%	0.03%	0.04%	0.05%	0.08%	0.04%	0.04%	0.03%	0.04%	0.03%	0.05%
Std	1.42%	2.09%	1.53%	1.38%	1.70%	1.00%	1.32%	1.12%	1.55%	1.44%	1.45%
Min	-11.66%	-22.49%	-14.74%	-12.04%	-14.87%	-9.87%	-12.06%	-10.38%	-13.55%	-17.44%	-11.97%
25%	-0.71%	-0.92%	-0.61%	-0.56%	-0.74%	-0.41%	-0.59%	-0.50%	-0.64%	-0.63%	-0.61%
50%	0.07%	0.10%	0.09%	0.09%	0.15%	0.06%	0.11%	0.06%	0.13%	0.10%	0.12%
75%	0.80%	1.02%	0.78%	0.73%	0.97%	0.54%	0.69%	0.60%	0.86%	0.75%	0.79%
Max	11.12%	14.87%	12.36%	11.91%	12.60%	8.17%	12.04%	7.42%	10.34%	8.42%	8.61%

3.1. Optimization Objective

The portfolio optimization goal is to maximize the Sharpe Ratio, where risk is measured by VaR instead of variance. Thus, the optimization problem is formulated to maximize the ratio of expected excess return to VaR. VaR was calculated using a delta-normal (parametric) approach. This assumes returns are normally distributed and uses the portfolio's mean and covariance to calculate VaR. For a one-day horizon and confidence level α (95% or 99%), the VaR is defined as the loss threshold such that the probability of a larger loss is 1- α. Mathematically, for a portfolio with daily mean return µp and standard deviation σ_pVaR is calculated as:

$$VaR_\alpha = -\mu_p + z_\alpha \sigma_p, \tag{16}$$

where zα is the α quantile of the standard normal distribution (e.g., z0.95 = 1.65, z0.99 = 2.33).

Therefore, the optimization objective is expressed as:

$$\max_{w} \frac{w^\top \mu - r_f}{-w^\top \mu + z_\alpha \sqrt{w^\top \Sigma w}}, \tag{17}$$

where w denotes the vector of portfolio weights, µ the vector of expected asset returns, Σ the covariance matrix of returns, rf the risk-free rate, and zα the critical value of the standard normal distribution corresponding to confidence level α.

The numerator $w^\top \mu - r_f$ represents the expected excess return of the portfolio, where $-w^\top \mu$ adjusts for mean return and $z_\alpha \sqrt{w^\top \Sigma w}$ scales the risk by the confidence quantile. For simplicity, the risk-free rate was set as zero. This formulation ensures that the optimization is explicitly aligned with downside risk consideration, rewarding portfolios that generate higher returns for a given VaR exposure.

3.2. Portfolio Construction

Using the formula described above, two optimized portfolios were constructed: 1) Opt95: Optimized to maximize $\frac{w^\top \mu - r_f}{VaR_{95\%}(w)}$. This portfolio internalizes the 95% VaR in its construction, effectively balancing expected return against the 95% downside risk; 2) Opt99: Optimized to maximize $\frac{w^\top \mu - r_f}{VaR_{99\%}(w)}$. This portfolio gives more weight to extreme-tail risk (99% VaR) in the optimization, likely leading to a more conservative allocation compared to Opt95. In practice, the implementation was carried out in Python by defining the function directly according to the formulas outlined in Section 3.1. No short selling is allowed (weights were constrained to [0,1]), and full investment was assumed (weights summing to 100%). The Equal-Weight Portfolio (EWP) was constructed by assigning $w_i = \frac{1}{11}$ to each sector ETF at the start, which was used as an undiversified benchmark to evaluate the performance of the optimized portfolios.

After obtaining the weight solutions, the overall VaR and return/VaR were calculated under a common confidence level at 95%. This ensures that the portfolio risk decomposition and performance metrics are evaluated consistently across all portfolios.

3.3. VaR Decomposition Metrics

Marginal VaR (MVaR), component VaR (CoVaR), and incremental VaR (IVaR) were employed together to attribute the total portfolio VaR to each asset. All the computations were realized in Python by defining the function using the formulas listed below.

3.3.1 Marginal VaR

MVaR estimates the changes in total VaR when the asset's position is infinitesimally increased. For asset i, $MVaR_i$ is the partial derivative of portfolio VaR with respect to a small change in the weight of i (formally, $MVaR_i = \partial VaR_p / \partial w_i$), so under the delta-normal model, MVaR was calculated as:

$$MVaR_i = \frac{\partial VaR_p}{\partial w_i} = z_\alpha \frac{\text{Cov}(R_i, R_p)}{\sigma_p} - \mu_i, \tag{18}$$

where R_i is the return of asset i, R_p is the portfolio return, σ_p is the portfolio volatility, μ_i is the expected return of asset i, and z_α is the critical value of the standard normal distribution. A higher $MVaR_i$ indicates that increasing the exposure to asset i will increase the overall portfolio VaR more significantly.

3.3.2 Component VaR

CoVaR represents the amount of the total VaR attributed to asset i, it can be decomposed into the sum of individual component contributions. In other words, CoVaR shows how much of the portfolio's tail risk comes from asset i. Larger values mean that the asset is a major driver of extreme losses. Here, it was obtained by scaling the marginal VaR with the asset's portfolio weight:

$$CVaR_i = w_i \cdot MVaR_i. \tag{19}$$

The percentage contribution of asset i to portfolio VaR was given by:

$$\%Contri_i = \frac{CVaR_i}{VaR_p}, \tag{20}$$

which will help identify which assets dominate portfolio risk.

3.3.3 Incremental VaR

IVaR measures the discrete change in total portfolio VaR if asset i is removed. Here, IVaR was evaluated by hypothetically removing each asset from the portfolio and recalculating the portfolio VaR.

So, VaR was calculated as:

$$IVaR_i = VaR_p - VaR_{p\setminus i}, \tag{21}$$

where $VaR_{p\setminus i}$ is the VaR of the portfolio without asset i. If $IVaR_i$ is positive, removing asset i reduces the VaR, so asset i was making the portfolio riskier - a net risk contributor. On the contrary, a negative $IVaR_i$ means that asset i was providing diversification benefits by reducing overall portfolio risk. Therefore, the sign and magnitude of IVaR help identify natural hedges versus risk amplifiers, as assets with negative IVaR would help stabilize the portfolio in extreme scenarios.

4. Empirical Results and Discussion

4.1. Portfolio Performance

Table 2 summarizes the portfolio performance of Equal-Weight Portfolios (EWP) and two optimized portfolios (Opt95 and Opt99). Figure 1 presents a clear comparison of final portfolio VaR and Return/Ratio Ratio between EWP and two optimized portfolios. As expected, both optimized portfolios consistently achieve higher Return/VaR ratios than EWP, representing a roughly 36% improvement in risk-adjusted performance. Although they both have slightly higher volatility and VaR, their expected returns increased more than enough to offset the higher risk. This indicates that the optimized portfolios generate substantially higher returns per unit of downside risk compared with the equal-weight benchmark.

Table 2. Portfolio Performance Summary

	Mean Return(%)	Volatility(%)	VaR(95%)	Return/VaR(95%)
EWP	0.0421	1.1983	0.0193	0.021829
Opt95	0.0596	1.2585	0.0201	0.029644
Opt99	0.0588	1.2427	0.0199	0.029617

Notes: Mean Return and Volatility are daily figures. VaR(95%) denotes the one-day VaR at 95% confidence level. Return/VaR(95%) is the ratio of Mean Return to 95% VaR. All values are rounded to four decimal places, except the Return/VaR(95%) ratios, which are given to six decimal places for precision.

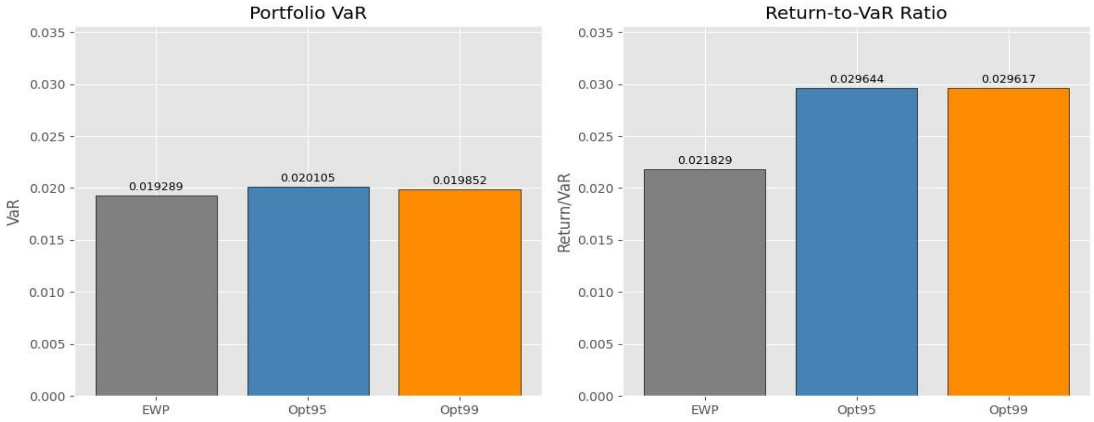

Figure 1. Comparison of Equal-Weight and Optimized Portfolio under 95% and 99% VaR constraints. The left panel reports the Portfolio VaR evaluated at 95% confidence level, while the right panel shows the Return-to-VaR ratio.

In comparison with Opt95, Opt99 has marginally lower volatility and VaR. However, its Return/VaR (95%) is essentially equal to Opt95, though an Opt99 strategy was expected to be more conservative, potentially sacrificing some expected return to reduce extreme-tail risk. A higher confidence level could penalize risky exposure more - the 99% VaR is roughly 1.41 times (2.33/1.65) than the 95% VaR, demanding more caution in asset selection and allocation. Indeed, the different confidence levels can lead to different portfolio ranking performance (Alexander & Baptista, 2003). A stricter VaR constraint will tilt the portfolio towards safety. Nonetheless, in our case, the trade-off was mild, so both optimized portfolios ended up similarly efficient. In sum, the optimization process successfully reallocated the portfolio to achieve greater return per unit of downside risk.

4.2. Allocation and Diversification

The different risk-return profiles of the portfolios are derived from the distinct weight allocations. Figure 2 shows pie charts of the asset weight allocation for the EPS, Opt95, and Opt99 portfolios, respectively. It is evident that the optimized portfolios are far from EWP. The equal-weight strategy invests uniformly across assets, meaning all the 11 sectors were evenly allocated (9.09%). In contrast, both VaR-optimized portfolios concentrated over 50% of the total weight in the technology sector (XLK), with supplementary allocations to consumer staples (XLP) and utilities (XLU), while excluding all other sectors (i.e., 0.00% weight). This is not surprising because assets that are attractive under a 95% VaR criterion often remain the same under a 99% VaR criterion. These assets are selected likely with either strong returns and acceptable risks or with diversification power that lowers portfolio VaR when combined. The subtle differences may include slightly higher weight on assets that are low risk to curtail the 99% tail risk, which was proved in the risk decomposition analysis.

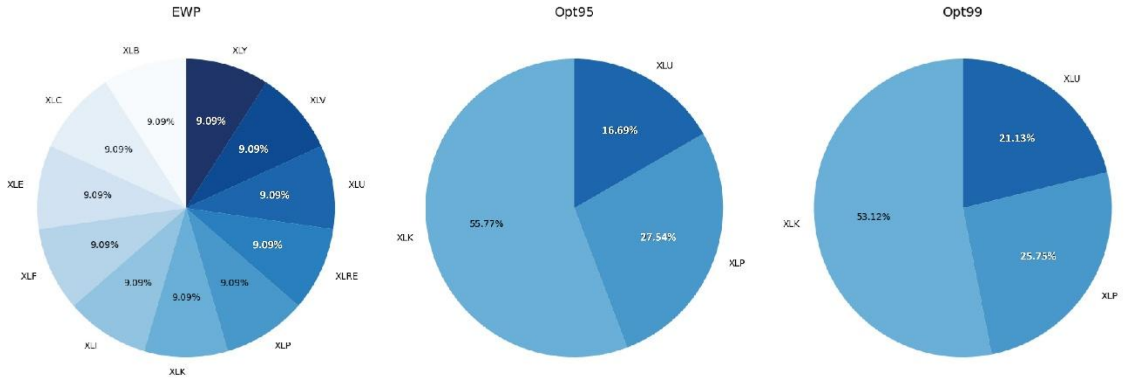

Figure 2. Asset allocations of the EPS, Opt95, and Opt99.

Specifically, in the Opt95 portfolio, XLK accounts for 55.77%, followed by XLP (27.53%) and XLU (16.69%). The Opt99 portfolio is similar, with slightly less dominated by XLK (53.12%), XLP (25.75%), and XLU (21.13%). In effect, the Opt99 portfolio could be seen as a "de-risked" version of Opt95. However, in our results, the difference was marginal, indicating that the chosen assets were consistently favourable under both risk levels. The allocation is reasonable as the technology sector often offers the highest excess return potential but also high volatility, whereas staples and utilities are more defensive and help offset the high risk brought by technology.

In sum, these concentrated allocations show that VaR-based optimization prefers allocations that maximize return per unit of downside risk, even if that means heavy concentration in a few assets.

The presence of staples and utilities in the optimized portfolios underscores that including some low-risk and defensive assets can allow the portfolio to take larger positions in high-return/high-risk assets without violating the VaR limit. In essence, the optimization seeks to find an efficient trade-off that sacrifices the diversification (focused on only three sectors) to achieve the best return per unit outcome.

4.3. Risk Decomposition and Contribution

A detailed VaR decomposition analysis of the portfolios (Tables 3, 4, and 5) provides further insights into how each asset contributes to the overall VaR (i.e., Portfolio risk). MVaR, CoVaR, and IVaR for each asset were analyzed, based on which we will understand which assets are the riskiest on the margin, how much each asset contributes to total VaR, and how the total VaR would change if an asset is removed.

For the EWP (Table 3), although each sector has the same position, its contribution to the total VaR spans from 5.81% to 11.49%. Also, the MVaR values differ across assets, reflecting different risk characteristics. Assets with the highest MVaR are the riskiest at the margin, assets with the lowest MVaR adds comparatively little risk, which are aligned with their highest CoVaR values and risk contribution. Higher-volatility sectors like energy (XLE), technology (XLK) naturally contributed more, whereas defensive sectors like utilities (XLU) and consumer staples (XLP) are relatively safe or diversifying. This means that even with equal weights, the risk contribution is not necessarily equally distributed. Therefore, diversification by capital does not translate to diversification by risk, which makes the importance of risk management self-evident.

Table 3. Risk Decomposition Results for Equal-Weight Portfolio

	Weight(%)	**MVaR**	**CoVaR(%)**	**Contribution(%)**	**IVaR**
XLE	9.0909	0.0244	0.2217	11.4913	0.000361
XLK	9.0909	0.0225	0.2049	10.6225	0.000264
XLF	9.0909	0.0222	0.2016	10.4514	0.000258
XLY	9.0909	0.0214	0.1943	10.0730	0.000164
XLB	9.0909	0.0208	0.1891	9.8054	0.000127
XLI	9.0909	0.0207	0.1877	9.7333	0.000119
XLRE	9.0909	0.0192	0.1741	9.0272	-0.000059
XLC	9.0909	0.0190	0.1728	8.9597	-0.000076
XLU	9.0909	0.0150	0.1363	7.0674	-0.000488
XLV	9.0909	0.0148	0.1343	6.9632	-0.000478
XLP	9.0909	0.0123	0.1120	5.8055	-0.000724

Notes: All values are rounded to four decimal places, except for IVaR, which is reported to six decimal places due to its very small magnitude.

The IVaR column (Table 3) provides additional insight. Positive IVaR means that removing those assets decreases the total VaR. In other words, those assets are net contributors to risk, which aligns with their high MVaR value - they are "risk drivers" in the portfolio. In contrast, assets having negative IVaR are risk-reducing assets by providing diversification benefits. From the EWP risk decomposition analysis (Table 3), it is clear that assets with positive IVaR are "risk hotspots", while others are risk mitigators. This explains the asset allocation in the optimized portfolios.

Table 4 shows the decomposition for the Opt95 portfolio. The weight distribution, as it was shown in Figure 2, is very concentrated: the technology sector (XLK) carries a 55.77% weight and contributes 71.50% of the total VaR, while consumer staples (XLP) at 27.54% weight contributes 16.69%, and utilities (XLU) at 16.69% weight contributes 11.81%. Almost all the portfolio's VaR comes from XLK. This allocation aligns with the finding above that XLK likely has the best return-to-risk trade-off. Also, XLK remains a positive IVaR value, whereas XLP and XLU have negative values. This means that XLK is still the major risk contributor in Opt95, XLP, and XLU play the diversifier roles.

In sum, the Opt95 portfolio's risk decomposition highlights a key point that it chose to tolerate concentrated risk in XLK because it offers a superior return-to-risk trade-off. XLP has the highest daily mean return of 0.08% with relatively high volatility of 1.70% (Table 1). XLP and XLU share a similar mean return of approximately 0.04%; the bigger proportion allocated to XLP is presumably due to it being the safest asset (lowest MVaRs and volatility). Comparing Opt99 to Opt95, the differences are minor (Table 5).

Table 4. Risk Decomposition Results for Optimized Portfolios at 95% Confidence Level

	Weight(%)	Active	MVaR	CoVaR(%)	Contribution(%)	IVaR
XLK	55.7742	Yes	0.0258	1.4375	71.5002	0.0033
XLP	27.5350	Yes	0.0122	0.3355	16.6890	-0.0033
XLU	16.6909	Yes	0.0142	0.2375	11.8109	-0.0014
XLY	0.0000	No	0.0210	0.0000	0.0000	0.0000
XLC	0.0000	No	0.0193	0.0000	0.0000	0.0000
XLF	0.0000	No	0.0184	0.0000	0.0000	0.0000
XLI	0.0000	No	0.0180	0.0000	0.0000	0.0000
XLB	0.0000	No	0.0178	0.0000	0.0000	0.0000
XLRE	0.0000	No	0.0172	0.0000	0.0000	0.0000
XLE	0.0000	No	0.0171	0.0000	0.0000	0.0000
XLV	0.0000	No	0.0140	0.0000	0.0000	0.0000

Notes: Active indicates whether the asset is included in this portfolio. Assets not included (Active = No) are displayed in ascending order of their MVaR values. All values are rounded to four decimal places.

The Opt99 gave a slightly higher weight to XLU, a relatively safer asset, and less to XLK, the riskiest asset, which is intuitive for a strategy that cares about extreme tail risk. This led to a marginal reduction in total VaR from 0.0201 (Opt95) to 0.0199 (Opt99). The risk contributions shifted such that XLU rose from 11.81% to 15.82%, while XLP decreased slightly from 16.69% to 16.02% and XLK from 71.50% to 68.16%. Figure 3 presents the risk contribution comparison between the two optimized portfolios.

Table 5. Risk Decomposition Results for Optimized Portfolios at 99% Confidence Level

	Weight(%)	Active	MVaR	CoVaR(%)	Contribution(%)	IVaR
XLK	53.1212	Yes	0.0255	0.013531	68.1580	0.0027
XLP	25.7451	Yes	0.0124	0.003180	16.0188	-0.0029
XLU	21.1338	Yes	0.0149	0.003141	15.8232	-0.0017
XLY	0.0000	No	0.0209	0.0000	0.0000	0.0000
XLC	0.0000	No	0.0192	0.0000	0.0000	0.0000
XLF	0.0000	No	0.0185	0.0000	0.0000	0.0000
XLI	0.0000	No	0.0180	0.0000	0.0000	0.0000
XLB	0.0000	No	0.0179	0.0000	0.0000	0.0000
XLRE	0.0000	No	0.0175	0.0000	0.0000	0.0000
XLE	0.0000	No	0.0172	0.0000	0.0000	0.0000
XLV	0.0000	No	0.0141	0.0000	0.0000	0.0000

Notes: Active indicates whether the asset is included in this portfolio. Assets not included (Active = No) are displayed in ascending order of their MVaR values. All values are rounded to four decimal places.

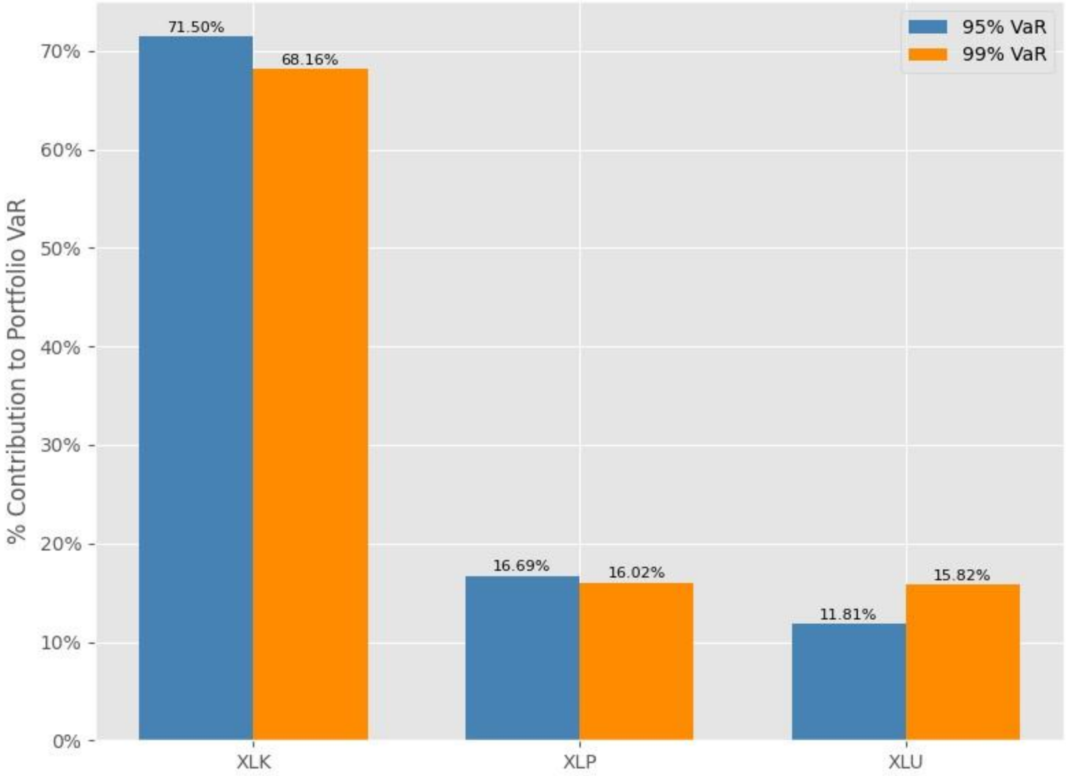

Figure 3. Risk Contribution Comparison under 95% and 99% VaR Confidence Levels.

In summary, from a risk management perspective, this concentrated risk profile means the optimized portfolio is heavily exposed to sector-specific shocks, notably in technology. The inclusion of staples and utilities provides some cushion, as evidenced by their diversifying effect. The risk decomposition confirms the intuitive trade-off anticipated that the VaR-optimized portfolios deliberately select a large exposure to the highest-return/high-risk sector, balanced by allocations to some low-risk sectors to control the overall VaR. Besides, the diversification also plays a key role in this optimization process by adding enough assets to maximize the return/VaR objective while satisfying the VaR constraint, which contrasts with the EWP's broad diversification.

5. Conclusion

This study demonstrated that VaR-based portfolio optimization can significantly enhance the risk-return profile relative to an equal-weight strategy. By explicitly incorporating a VaR constraint, the optimized portfolios achieved higher returns. The return/VaR maximization approach tends to concentrate assets that offer the best trade-off between expected return and VaR. In this study, a large allocation was attributed to the technology sector, mediated by position in defensive sectors (i.e., staple and utility) to satisfy the risk limits. As a result, compared to the equal-weight benchmark, the optimized portfolios delivered superior performance - higher mean returns and return/VaR ratios.

Two optimized portfolios produced comparable results. A stricter 99% VaR constraint led to a slightly more conservative portfolio, with marginally lower returns and risk, but largely the same asset allocation. In other words, increasing the confidence level from 95% to 99% did not fundamentally change the portfolio's structure but the proportions. The 99% portfolio is more towards a higher allocation in safe assets of XLU, as it demands protection against more extreme tail events, thus by sacrificing some return to control risks. This implies that one should set the VaR confidence level in accordance with one's risk tolerance; a higher level (e.g., 99% or beyond) will emphasize stability at the cost of return, whereas a lower level (e.g., 95% or below) allows more return-seeking at the expense of accepting a higher potential loss in extreme cases. However, in this case, the difference was small, likely because the assets did not have behavior in the 99% tail compared to the 95% tail under the normality assumption.

The risk decomposition provides a valuable insight into understanding how the optimized portfolios manage risk. A large portion of the total VaR was attributed to a high-return/high-risk asset (technology, XLK), which is a natural consequence of optimizing the return/VaR ratio. However, the inclusion of other assets with different risk levels is crucial to keep the overall VaR within acceptable bounds - a clear demonstration of diversification's role in a concentrated portfolio. Notably, the other two assets, XLU and XLP, with a negative IVaR value, indicate their inclusion reduced the portfolio's total risk. From a risk management perspective, the decomposition analysis is invaluable, as it allows investors to identify the main drivers of downside risk and confirm whether the portfolio is relying on a broad diversification. In optimized portfolios, it clearly shows that a concentration in one aggressive asset, supplemented by two smaller defensive assets, is used to mitigate the potential risks. In contrast, the EWP allowed the assets to contribute a lot of VaR without sufficiently higher returns, essentially diluting its risk-adjusted performance.

In summary, the VaR-based optimization strategy proved effective in enhancing performance, but it also led to concentrated risk exposures. While VaR optimization aligns the portfolio with a desired risk level, the resulting concentration means that the portfolio could be more vulnerable to unexpected shocks in the heavily weighted sectors. For investors, these results highlight that, though a higher risk-adjusted return and clear focus on downside risk, it demands careful oversight of the concentrated positions. An equal-weight portfolio strategy, by contrast, spreads risk more evenly but forgoes some return potential. In practice, portfolio managers might impose additional constraints or modify certain parameters to avoid such an extreme portfolio allocation. However, the adoption of such an approach largely depends on the investors' comfort with the resulting portfolio profile. By refining the method and rigorously testing under various conditions, one can harness the benefits of VaR-based optimization while mitigating its drawbacks, contributing to more effective risk-aware investment strategies. Therefore, based on current findings, in the future, other methods are worth exploring, for example, the conditional VaR-based (CVaR) optimization. Optimizing portfolios using CVaR could yield different allocations, potentially more diverse.

In conclusion, VaR-based portfolio optimization offers an alternative to mean-variance optimization for investors, particularly concerned with downside risk. The methodology and insights here are valuable for risk management, as they present how VaR can be used not just as a constraint but as a driver of portfolio decisions, and how decomposing VaR helps in understanding those decisions. Risk decomposition into MVaR, CoVaR, and IVaR proved to be a powerful tool to validate the portfolio's risk structure - it confirmed that risk was allocated to the intended assets and quantified how each position would impact the portfolio's tail risk. Portfolio managers can use these insights to understand where risk is coming from and which positions are truly adding value in an optimized portfolio. Ultimately, the combination of Return/VaR optimization and VaR decomposition facilitates a performance-focused portfolio construction process. It exemplifies how modern risk management techniques can be integrated into portfolio optimization to achieve better outcomes, meeting the goals of high returns and controlled downside risk.

Data Availability Statement

The data used in this study are derived from publicly available sources and can be accessed via Yahoo Finance.

Funding

This research received no external funding.

Acknowledgements

The author would like to thank Dr. David Gabauer for his valuable guidance and constructive feedback during the development of this research.

Declaration of the Use of Generative AI

The author declares that no generative AI tools were used in the preparation of this manuscript.

Conflicts of Interest

The author declares that there are no conflicts of interest related to this study.

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